Mathematical Products

- Multiplication of two scalars: Result – scalar
  Example: $m_1 = 3\text{kg}$, $m_2 = 5m_1 = 15\text{kg}$
- Multiplication of a scalar and a vector: Result – vector
  Example: $\mathbf{v_1} = 3\text{m/s}\ \mathbf{i}$, $\mathbf{v_2} = 5\mathbf{v_1} = 15\text{m/s}\ \mathbf{i}$
- Multiplication of two vectors: Result – Either scalar or vector
  Scalar Product: $R = \mathbf{A} \bullet \mathbf{B}$
  Vector Product: $\mathbf{R} = \mathbf{A} \times \mathbf{B}$
- How do we define how to get R and **R**?
  Let's deal with the Vector Product later!

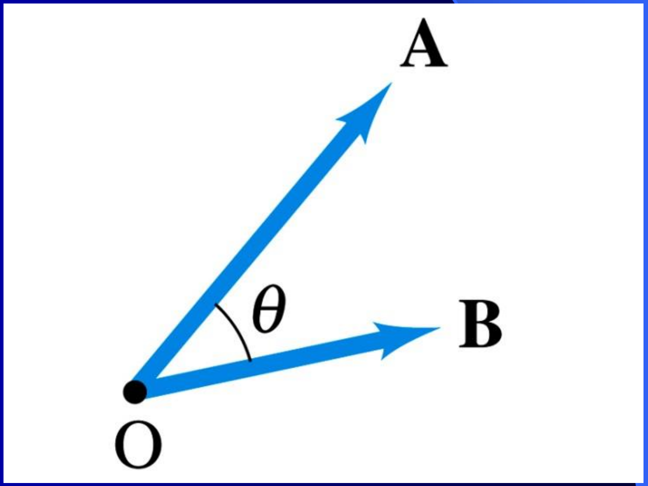
Scalar Products

- Scalar Product: $R = \mathbf{A} \bullet \mathbf{B} = AB \cos\theta$
- $\mathbf{A} \perp \mathbf{B} \rightarrow \mathbf{A} \bullet \mathbf{B} = 0$ $\mathbf{A} \parallel \mathbf{B} \rightarrow \mathbf{A} \bullet \mathbf{B} = AB$

- Product of A with component of B in direction A

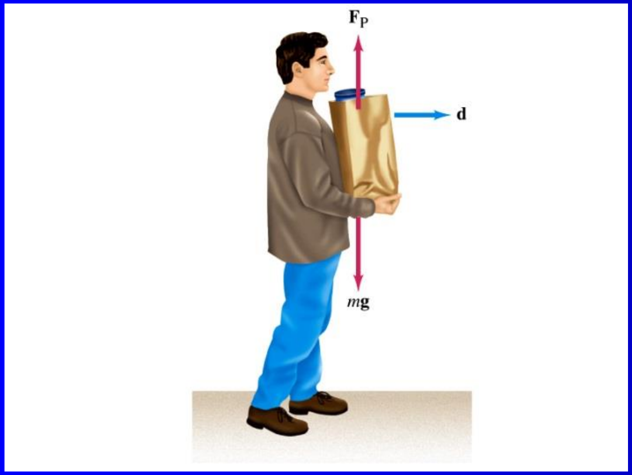
- $\mathbf{A} = A_x\, \mathbf{i} + A_y\, \mathbf{j} + A_z\, \mathbf{k}$
- $\mathbf{B} = B_x\, \mathbf{i} + B_y\, \mathbf{j} + B_z\, \mathbf{k}$
- $\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$

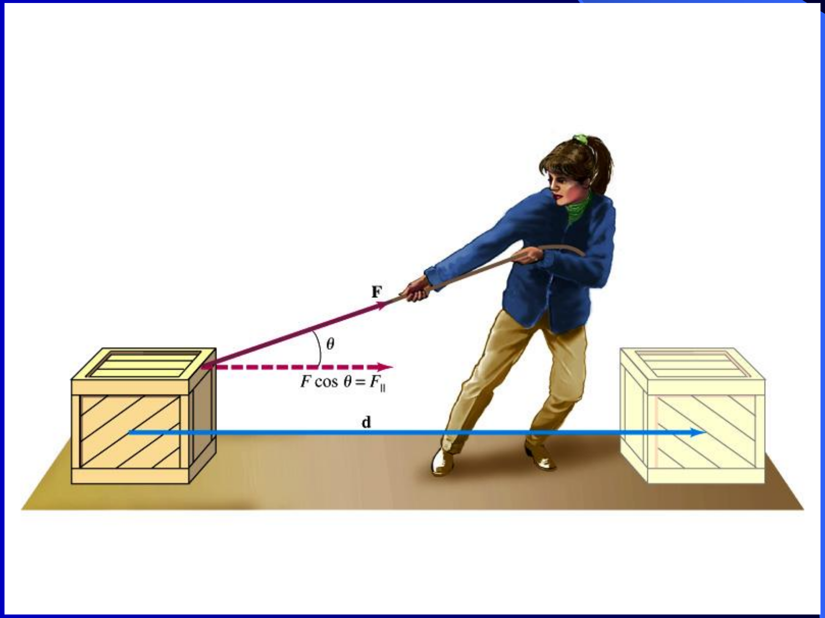
- $\mathbf{i} \bullet \mathbf{i} = \mathbf{j} \bullet \mathbf{j} = \mathbf{k} \bullet \mathbf{k} = 1$
- $\mathbf{i} \bullet \mathbf{j} = \mathbf{i} \bullet \mathbf{k} = \mathbf{j} \bullet \mathbf{k} = 0$



Work done by a Constant Force

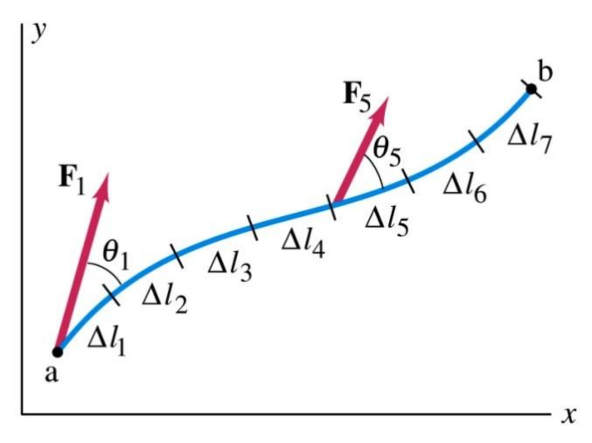
- Scalar Product: $W = \mathbf{F} \bullet \mathbf{d} = Fd\cos\theta = F_d\, d$
- If you don't displace the object, you don't do work
- Unit: N m = J

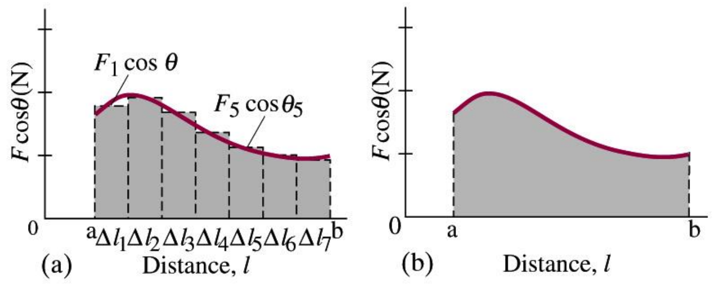




Work done by a Varying Force

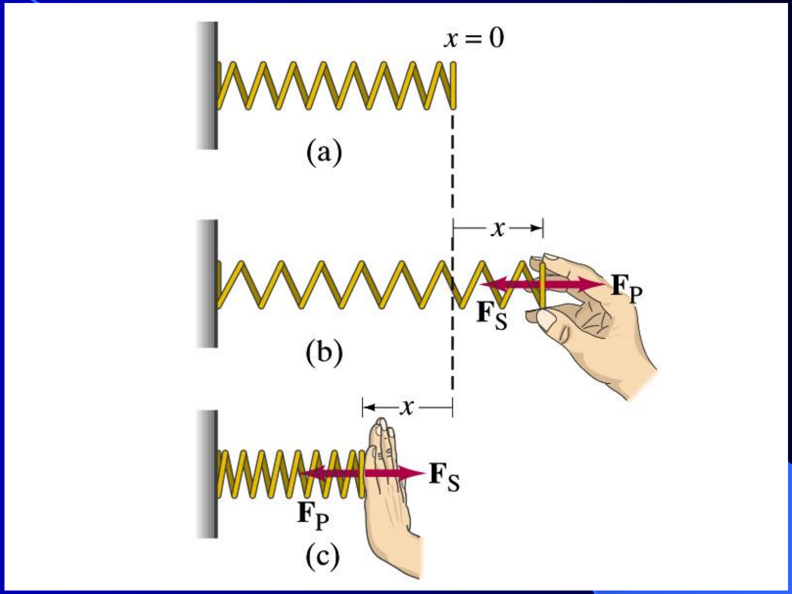
- $\Delta W_1 \approx F_1 \cos\theta_1 \, \Delta l_1$
  $\Delta W_5 \approx F_5 \cos\theta_5 \, \Delta l_5$
- $W \approx \Sigma F_i \cos\theta_i \, \Delta l_i$
- $W = \lim_{\Delta l_i \to 0} \Sigma F_i \cos\theta_i \, \Delta l_i$
  $(= \int F\, dl)$
- $(W = \int F_x \, dx + \int F_y \, dy + \int F_z \, dz)$

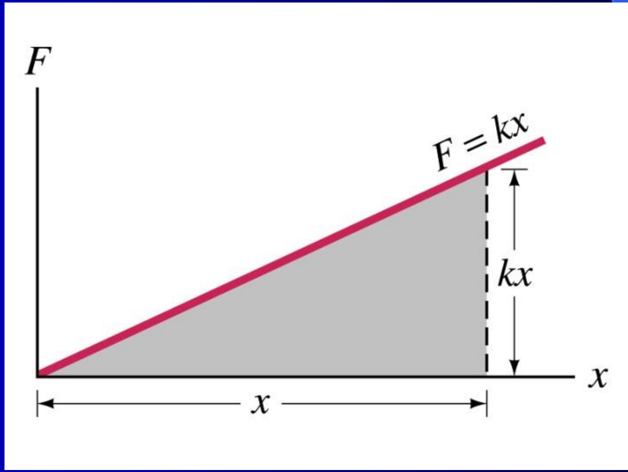




E.g. a one-dimensional spring

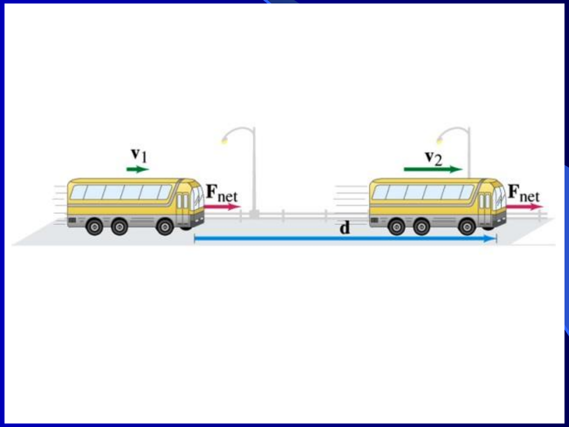
- $F_P (x) = k x$
- $F_S (x) = - k x$
- Look at area of triangle:
  $W_P = ½ x kx = ½ k x^2$





Kinetic Energy

- Think of an object accelerating from v_1 to v_2 (a=const.)
- const. = F = ma, W = Fd



- remember: $v_2^2 = v_1^2 + 2ad$
  $a = (v_2^2 – v_1^2) / 2d$
- $W = Fd = mad = md\,(v_2^2 – v_1^2) / 2d = ½\, m\, v_2^2 – ½\, m\, v_1^2$
- $K = ½\, m\, v^2$ $W = K_2 – K_1 = \Delta K$
- Work-Energy Principle: The net work done on an object is equal to the change in its kinetic energy