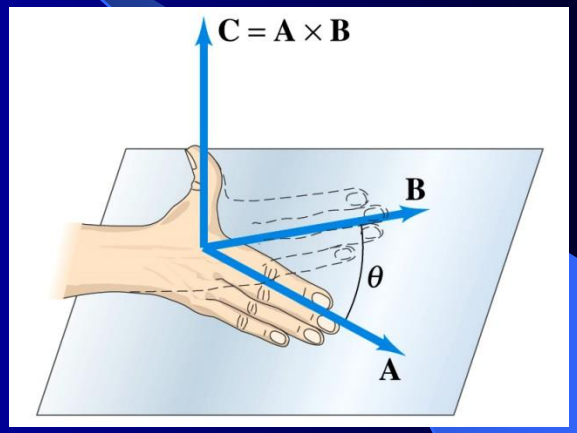
Vector (Cross) Product

- Vector Product should have these properties:
  - $C = \left|\vec{A} \times \vec{B}\right| = AB\sin\theta$
  - $\vec{C}$ *perpendicular to* $\vec{A}, \vec{B}$



- This definition achieves that:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = A_y B_z \vec{i} - A_z B_y \vec{i} + A_z B_x \vec{j} - A_x B_z \vec{j} + A_x B_y \vec{k} - A_y B_x \vec{k}$$

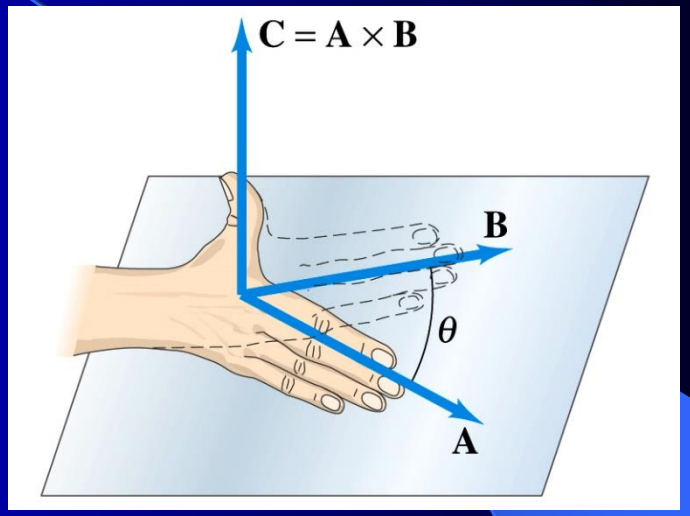
Properties of Vector Product

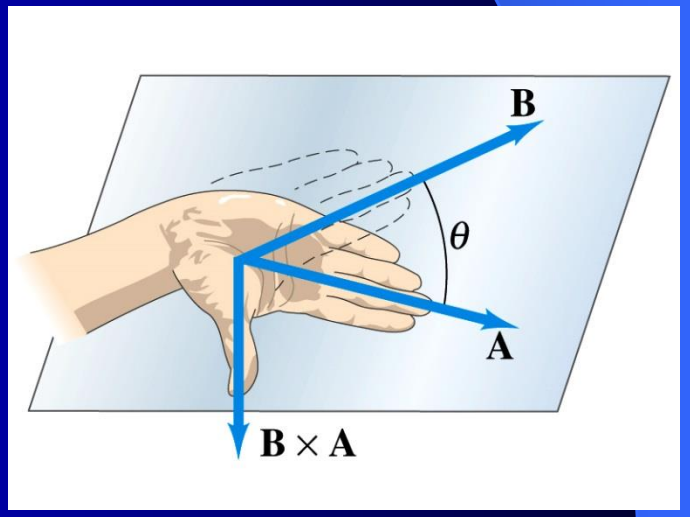
$$\vec{A}\times\vec{A}=0$$

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

$$\vec{A}\times\left(\vec{B}+\vec{C}\right)=\left(\vec{A}\times\vec{B}\right)+\left(\vec{A}\times\vec{C}\right)$$

$$\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$$





Vector Product, e.g. Torque

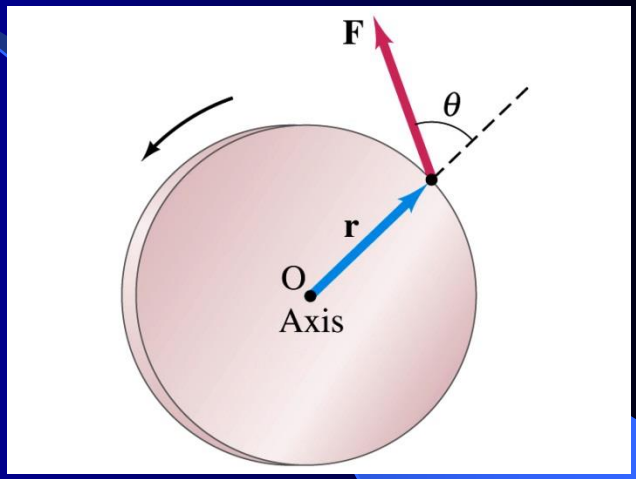
$$\tau = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta$$

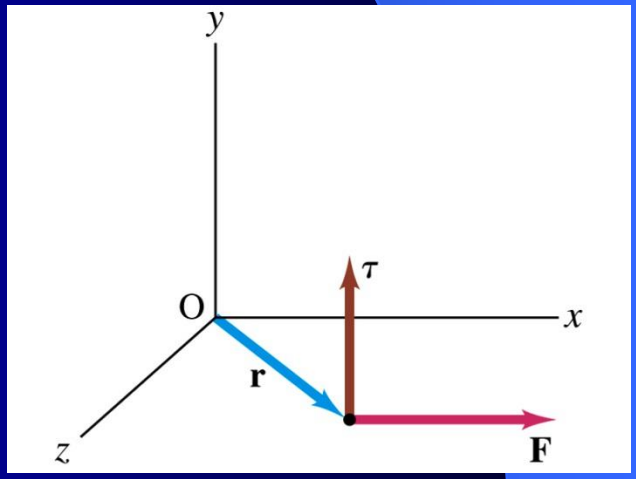
$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Ex. $\vec{r} = (1.2m)\vec{i} + (1.2m)\vec{k}$

$$\vec{F} = (150N)\vec{i}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.2m & 0 & 1.2m \\ 150N & 0 & 0 \end{vmatrix} = 0\vec{i} + (180mN)\vec{j} + 0\vec{k}$$

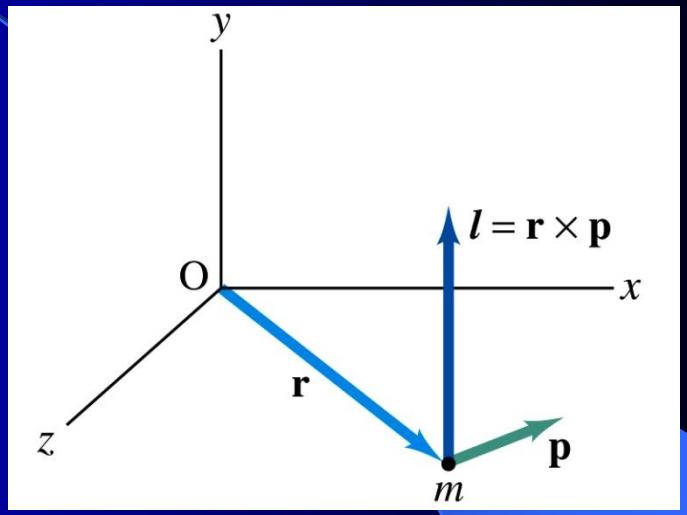




Angular Momentum

$$L = |\vec{r} \times \vec{p}| = rp\sin\theta$$

$$\vec{L} = \vec{r} \times \vec{p}$$



- Example

$$L = |\vec{r} \times \vec{p}| = rp\sin 90^\circ = rmv$$